Gaussian processes for modelling stellar activity and detecting planets

Vinesh Rajpaul[1], **Suzanne Aigrain**[1], **Stephen Roberts**[2]

[1] Oxford Astrophysics
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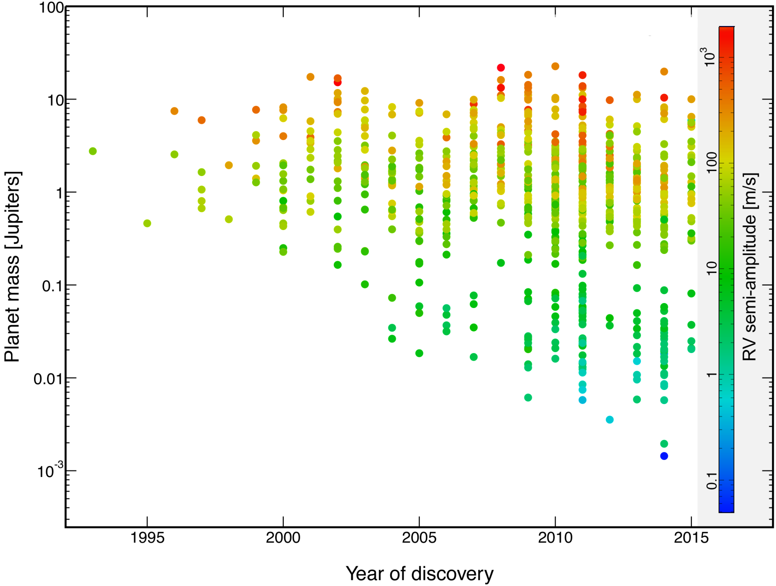
31 March 2016

Doppler spectroscopy and exoplanets



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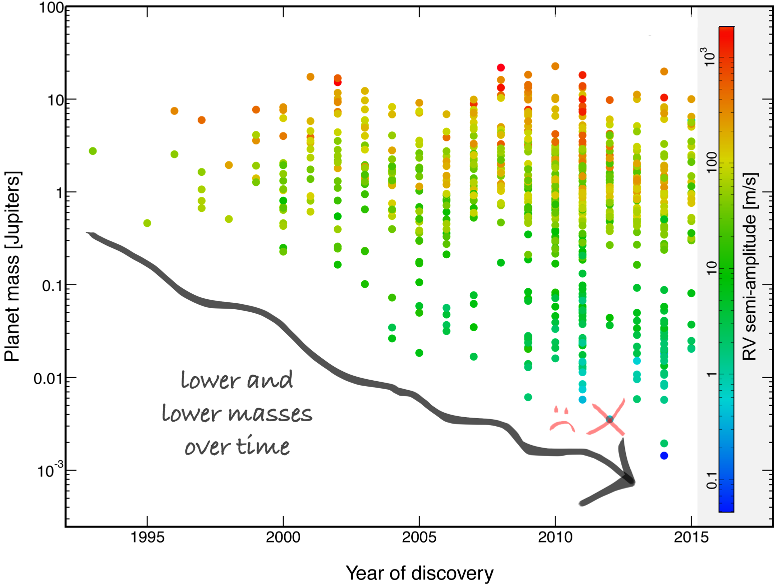

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- Longer time-scale signals (days to years) associated with **rotationally-modulated activity**

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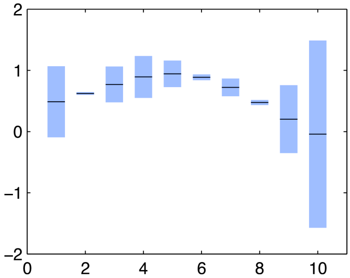
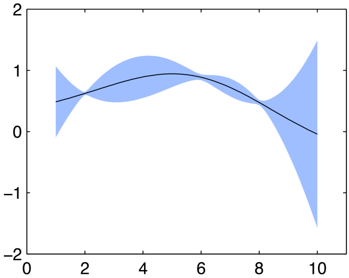
- Characterised by **time-scales similar to those associated with planets** (days to years)
- **Quasi-periodic** (periodic stellar rotation + evolving active regions + long-term activity cycles)
- Characterised by some **degree of smoothness** (active regions don't change instantaneously)

So what **is** a Gaussian process?

(And why should we care?)

Introducing GPs





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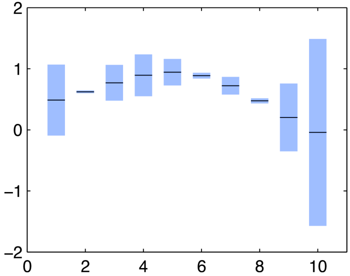

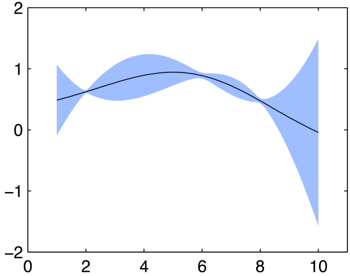

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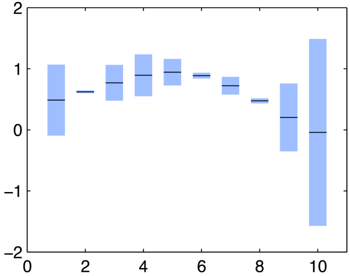

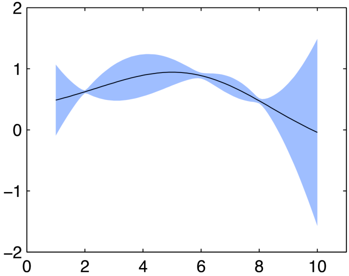

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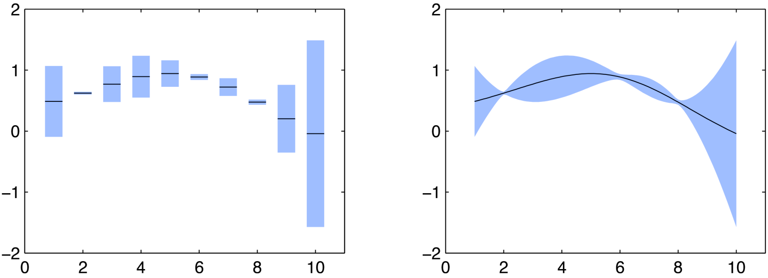

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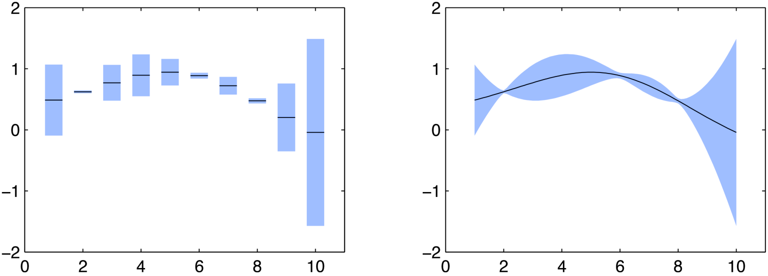

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- Fully specified by a **mean function** and **covariance function** (cf. mean vector, covariance matrix)
- In practice, we **parametrise** the mean and covariance functions (instead of writing down an infinite number of values!)

Why GPs?

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- Flexible yet principled, data-driven way to perform **Bayesian inference about functions** → rigorous treatment of uncertainty, model comparison, etc.
- Obey lots of **convenient analytical identities**
- Can **model functions by parametrising covariance between data points** → signal variance, evolution time-scales, (quasi)periodicities, smoothness, noise levels, stationarity, isotropy, etc.

Towards GP regression

$$y_i = f(t_i) + \varepsilon_i \quad , f \sim \mathcal{GP}(\mathbf{m}, \mathbf{K})$$

$$\mathbf{m} = m(\mathbf{t}; \phi)$$

mean function
deterministic components
(e.g. planets)

$$\mathbf{K}_{ij} = k(t_i, t_j; \theta)$$

covariance function
stochastic processes/stuff we can't
parametrise directly (e.g. activity)

hyper-parameters

$$\log \mathcal{L}(\theta, \phi) \propto (\mathbf{y} - \mathbf{m})^{\mathrm{T}} \mathbf{K}^{-1} (\mathbf{y} - \mathbf{m}) \quad + \log \det \mathbf{K}$$

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Example: quasi-periodic covariance function

$$k(\tau) \propto \exp\left(\frac{-\tau^2}{2\lambda_e^2}\right)\exp\left(\frac{-\sin^2(\pi\tau/P)}{2\lambda_P^2}\right),$$

evolutionary time-scale (λ_e)

roughness/ structure per period (λ_P)

characteristic period (P)

where $\tau := t - t'$

time between any 2 points

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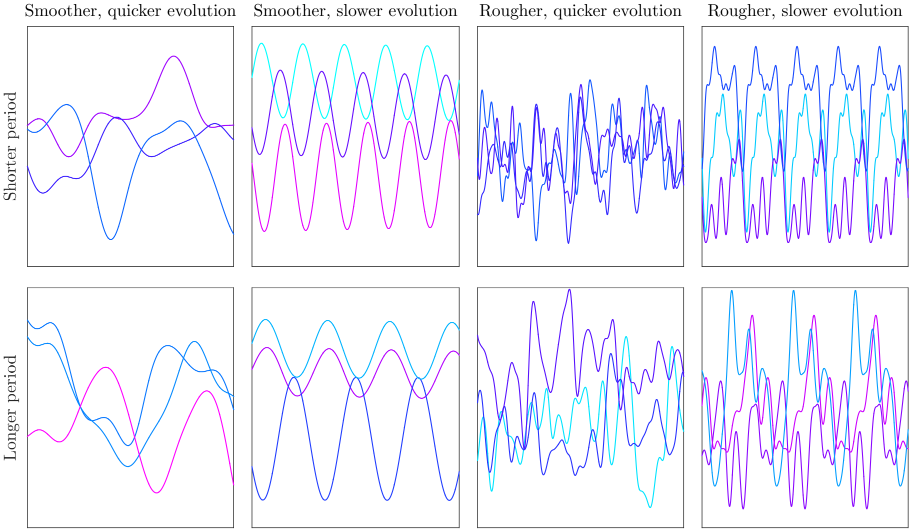
Function draws: quasi-periodic covariance function



Why not a parametric model?

[...] fitting sine waves at the rotational period of the star and the significant harmonics [...]

The global model fitted on the RVs is therefore:

$$
\begin{aligned}
\text{subset 2008} \quad : \quad & lin0 + lin1 \cdot JDB_{2008} + lin2 \cdot JDB_{2008}^2 + A_{RV-Rhk} \cdot RHK_{low\,freq,2008} \\
\text{subset 2009} \quad : \quad & lin0 + lin1 \cdot JDB_{2009} + lin2 \cdot JDB_{2009}^2 + A_{RV-Rhk} \cdot RHK_{low\,freq,2009} \\
& + A11s \cdot sin(\frac{2\pi}{P1}) \cdot JDB_{2009} + A11c \cdot cos(\frac{2\pi}{P1}) \cdot JDB_{2009} \\
& + A12s \cdot sin(\frac{2\pi}{P1/2}) \cdot JDB_{2009} + A12c \cdot cos(\frac{2\pi}{P1/2}) \cdot JDB_{2009} \\
\text{subset 2010} \quad : \quad & lin0 + lin1 \cdot JDB_{2010} + lin2 \cdot JDB_{2010}^2 + A_{RV-Rhk} \cdot RHK_{low\,freq,2010} \\
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→ 23 free parameters (before adding a planet)

Activity model for Alpha Cen B, from Dumusque *et al.* (2012)

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Recap

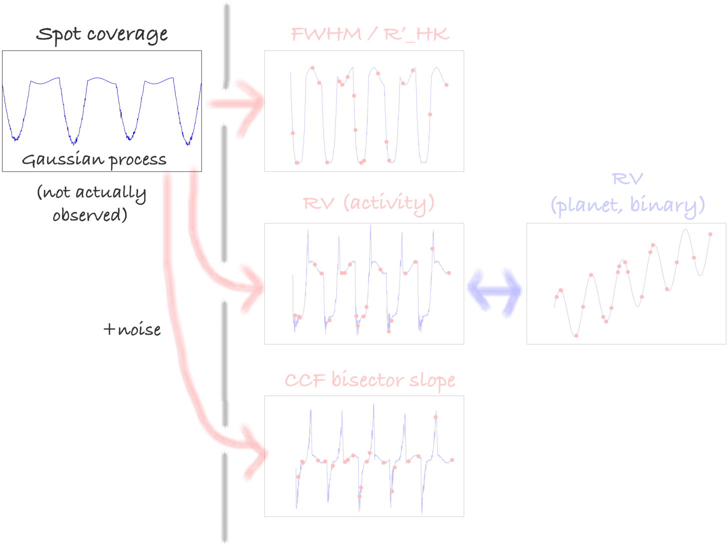
GPs $\rightarrow$ easy Bayesian inference about functions

Now, on to the science!

① GPs to **disentangle** activity and planetary signals

a.k.a. model ALL available data

Disentangling activity and planetary signals



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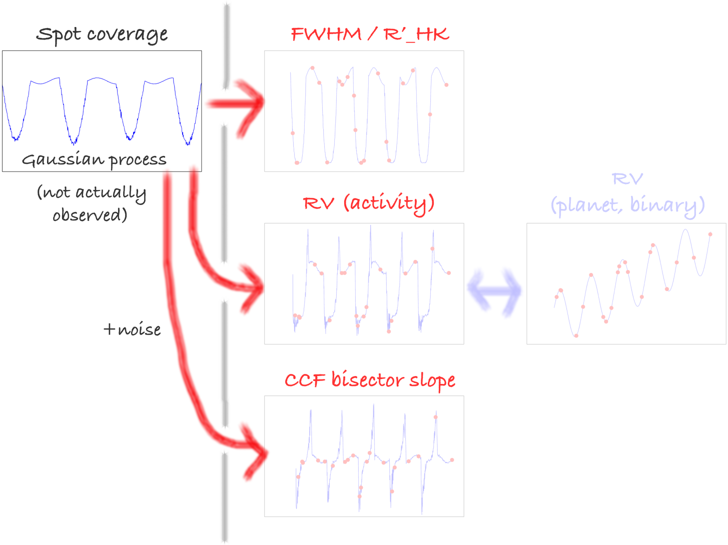

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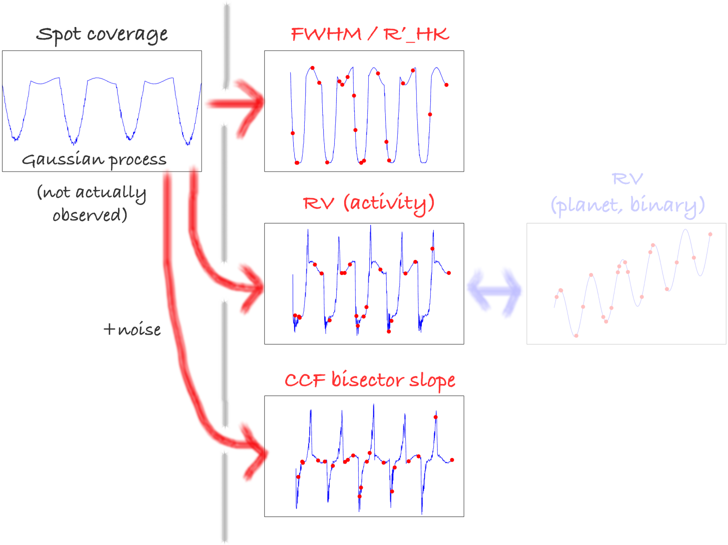

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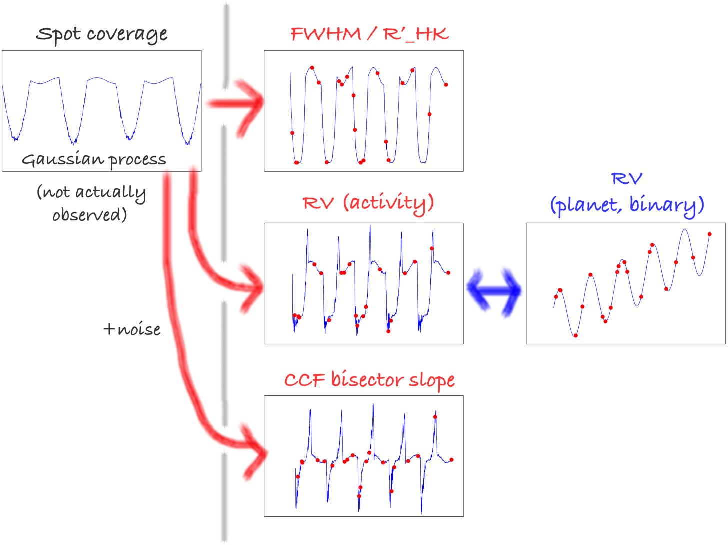
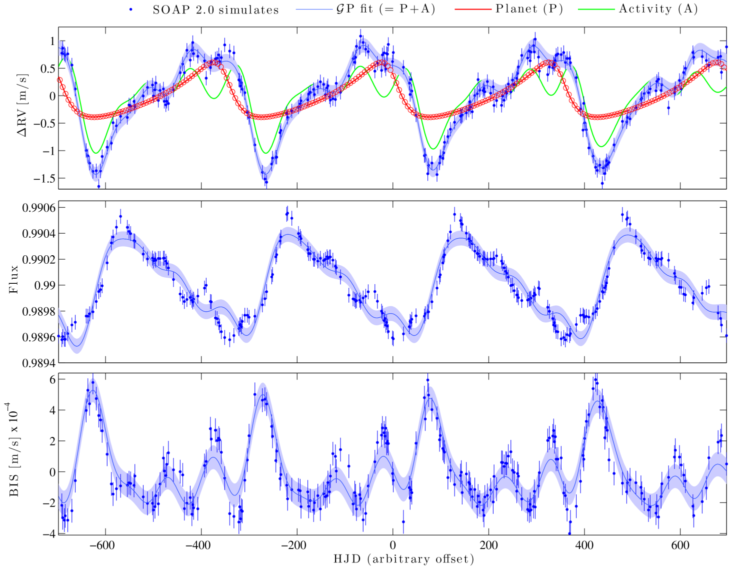


It works as intended



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- $\rightarrow$ Can use e.g. the FF' method, which says $\Delta \mathrm{RV}$ related to photometric flux and its first derivative (Aigrain *et al.*, 2012, MNRAS)
- More details on the physics:

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **452,** 2269–2291 (2015) doi:10.1093/mnras/stv1428

A Gaussian process framework for modelling stellar activity signals in radial velocity data

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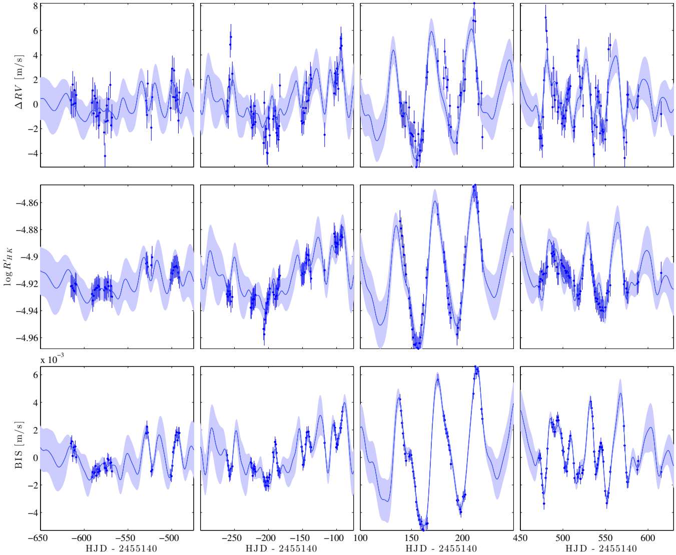
[2] *Pattern Recognition and Machine Learning Group, Department of Engineering Science, University of Oxford, Oxford OX1 3PJ, UK*

Accepted 2015 June 24. Received 2015 June 22; in original form 2015 March 16

Some science results

1. Proof of concept: accurately **recover already-published exoplanet parameters**, e.g. for Gliese 15 A, CoRoT-7 (Rajpaul *et al.*, 2015; MNRAS)
2. New result: **discovery of HD175607 b**, most metal-poor G dwarf with an orbiting sub-Neptune; $P_{\mathrm{star}} \approx P_{\mathrm{planet}}$ (Mortier *et al.*, 2015; A&A)
3. New result: **demonstration that Alpha Cen Bb is a false positive** (Rajpaul *et al.*, 2016; MNRAS Letters)

Four seasons of Alpha Cen B data



② GPs as a powerful **simulation tool**

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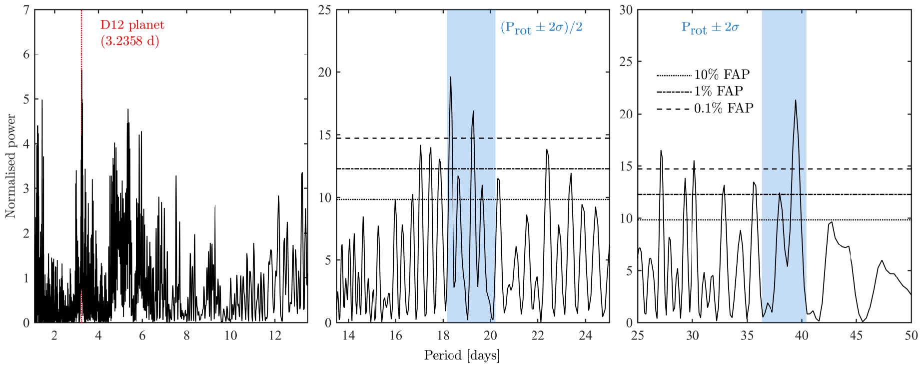
The example of Alpha Cen B

Power spectrum of the window function

(Rajpaul et *al.*, 2016; MNRAS Letters)



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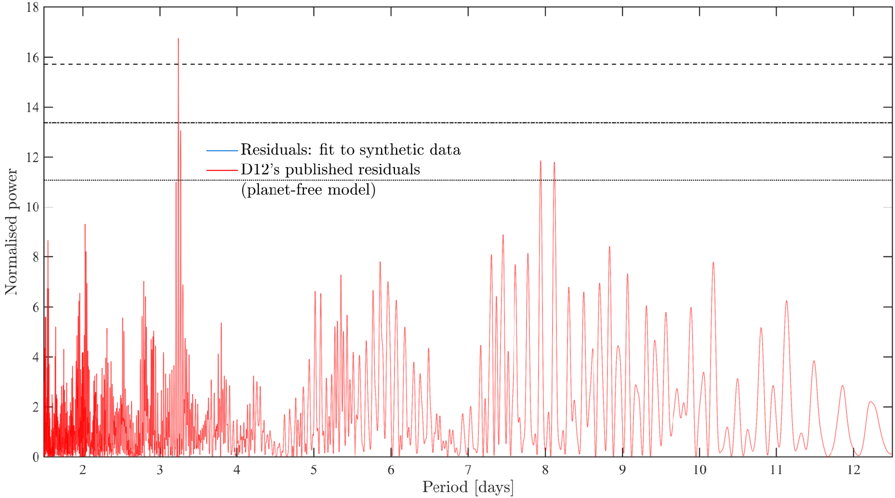

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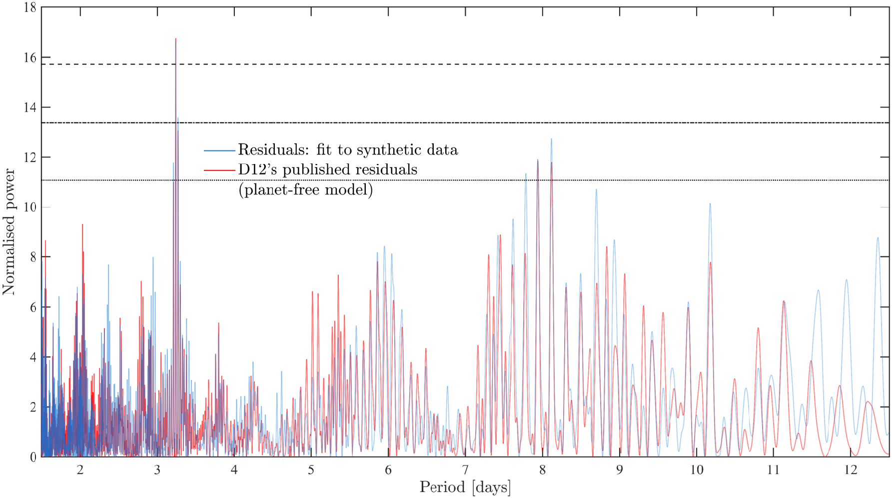

③ GPs to study **periodic phenomena**

a.k.a. we can do better than Lomb-Scargle periodograms

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Beyond the Lomb-Scargle periodogram

- Lomb-Scargle method/LSSA is convenient, easy to understand, and popular...but restrictive, and often inappropriate
- What about signals that are non-sinusoidal, quasi-periodic, contain correlated noise, etc...?
- Drop-in replacement: GP periodogram (with Angus *et al.*)

Quasi-periodic covariance function (again)

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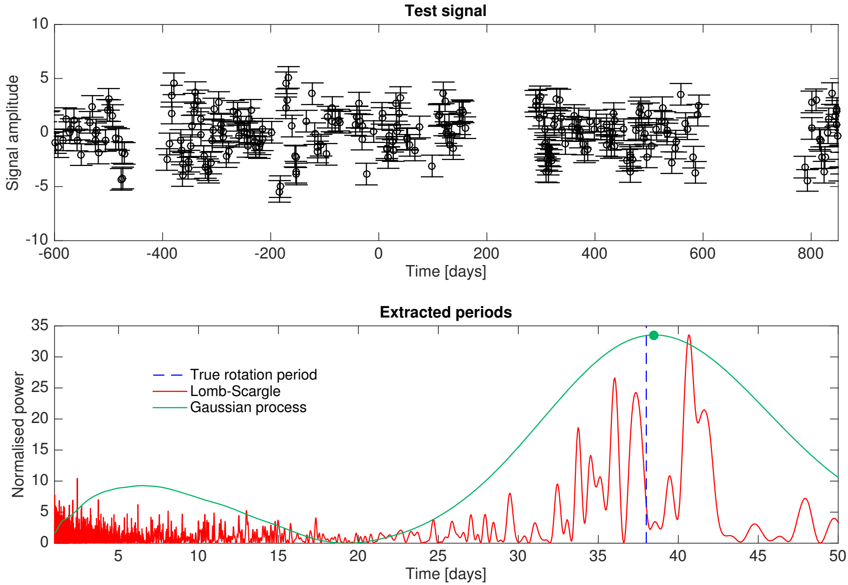
GP periodogram



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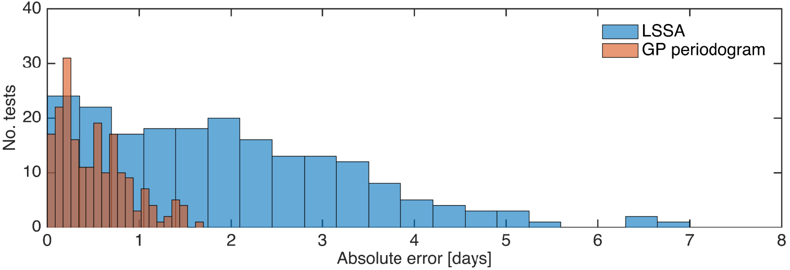

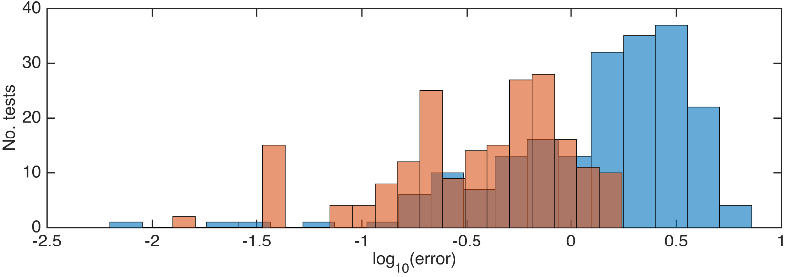

One step-further: differential rotation

- Assume distribution of quasi-periodic signals, each generated by quasi-periodic kernel. Say $f(P') \sim \mathcal{N}(P, \sigma)$, or $f(P') \sim \mathcal{U}(P - \sigma, P + \sigma)$

- Integrate over this distribution to get new covariance kernel

- Characterise differential rotation via posterior distribution of P and σ

- Early results are promising...watch this space (injection tests currently underway)

④ GPs to extract **precise RVs** directly from spectra

towards < 10 cm/s precisions ?

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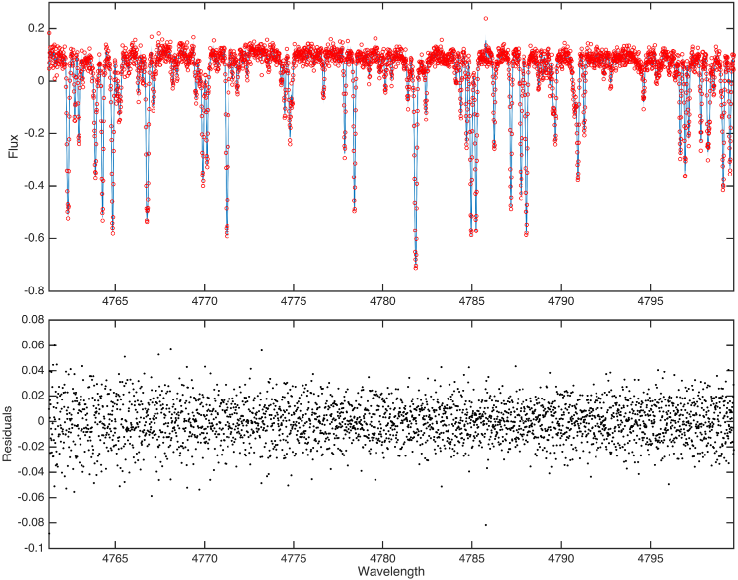
- Treat the GP models as **noise-free interpolants** of the spectra
- Fit $(N-1)$ RV shifts/translations to **maximise alignment**
- Optional: **unshift** all spectra to obtain high resolution "master" spectrum

Ultra high-precision RVs



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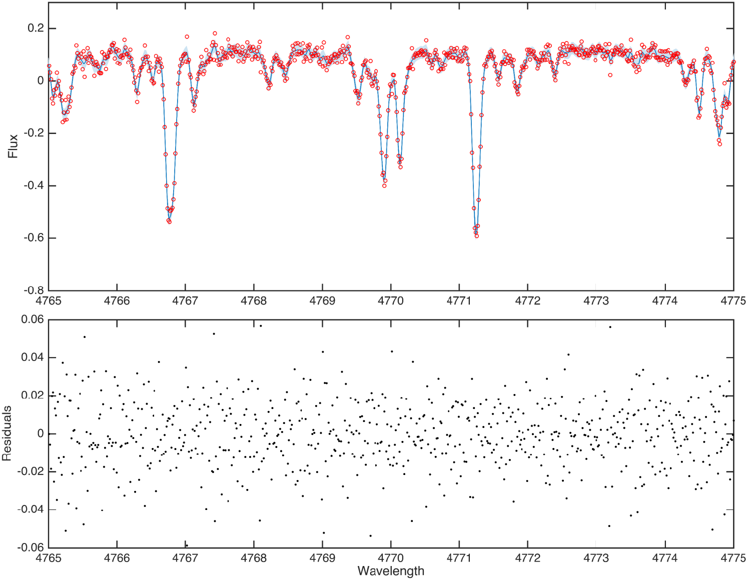

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- Beyond RVs: extract better measures of stellar activity?

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- Useful addition to the toolbox of anyone trying to detect or characterise exoplanets
- Some recent applications on which I've worked
    1. GPs to disentangle activity and planets
    2. GPs as a powerful simulation tool
    3. GPs to characterise periodic phenomena e.g. stellar rotation
    4. GPs for extracting high-precision RVs from spectra

Thanks!
Any questions?

get in
touch

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